Calculus Warm Up #2-2

Try to come up with an easy way to do this problem. A little boy named Gauss did. He was sent to the corner because he was misbehaving and told he couldn't come out until he had the answer. It took him only a few seconds, and he didn't have a calculator!

$1 + 2 + 3 + \ldots + 98 + 99 + 100 =$

$1 + 2 + 3 + \ldots + 98 + 99 + 100 =$

$+100$, $+99$, $+98$, $+3$, $+2$, $+1$

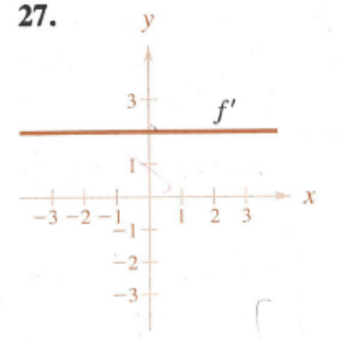
101, 101, 101, 101, 101, 101

$$\frac{100(101)}{2} = 50(101)$$

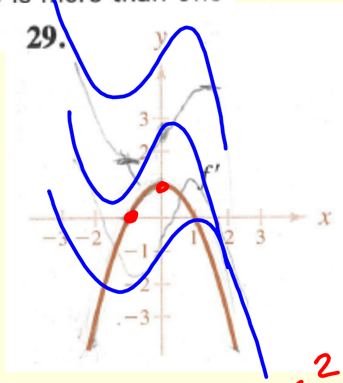
HW Questions: p. 238

In Exercises 27–30, the graph of the derivative of a function is given. Sketch the graphs of *two* functions that have the given derivative. (There is more than one correct answer.)

27.



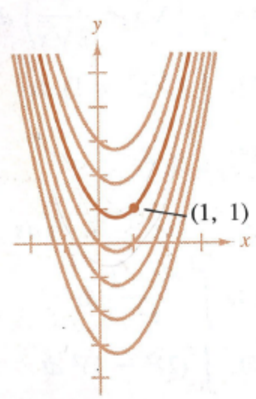
29.



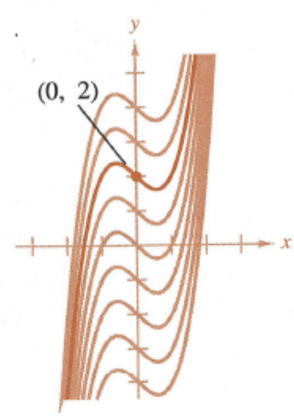
$y = x^2 + 1$

In Exercises 31–34, find the equation of the curve, given the derivative and the indicated point on the curve.

31. $\dfrac{dy}{dx} = 2x - 1$



33. $\dfrac{dy}{dx} = 3x^2 - 1$



In Exercises 35–38, find $y = f(x)$ satisfying the given conditions.

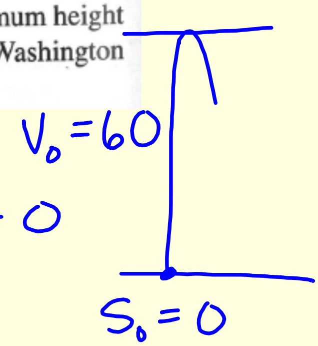
35. $f''(x) = 2, f'(2) = 5, f(2) = 10$
36. $f''(x) = x^2, f'(0) = 6, f(0) = 3$
37. $f''(x) = x^{-3/2}, f'(4) = 2, f(0) = 0$
38. $f''(x) = x^{-3/2}, f'(1) = 2, f(9) = -4$

In Exercises 39–43, use $a(t) = -32$ ft/s^2 as the acceleration due to gravity. (Neglect air resistance.)

39. An object is dropped from a balloon that is stationary at 1600 feet above the ground. Express its height above the ground as a function of t. How long does it take the object to reach the ground?

40. A ball is thrown vertically upward from the ground with an initial velocity of 60 feet per second. How high will the ball go?

41. With what initial velocity must an object be thrown upward (from ground level) to reach a maximum height of 550 feet (approximate height of the Washington Monument)?



$s(t) = -16t^2 + 60t + 0$

$v(t) = -32t + 60$

$0 = -32t + 60$

42. Show that the height above the ground of an object thrown upward from a point s_0 feet above the ground with an initial velocity of v_0 feet per second is given by the function

$$f(t) = -16t^2 + v_0 t + s_0.$$

43. A balloon, rising vertically with a velocity of 16 feet (V_0) per second, releases a sandbag at the instant when the balloon is 64 feet (S_0) above the ground.

(a) How many seconds after its release will the bag strike the ground?

(b) With what velocity will it reach the ground?

find t when $s(t) = 0$

$s(t) = -16t^2 + 16t + 64$

44. Assume that a fully loaded plane starting from rest has a constant acceleration while moving down the runway. Find this acceleration if the plane requires, on the average, 0.7 miles of runway and a speed of 160 miles per hour before lifting off.

Even answers follow.

36) $f(x) = \frac{x^4}{12} + 6x + 3$

38) $f(x) = 4x - 4\sqrt{x} - 28$

or $f(x) = -4(\sqrt{x} - x + 7)$

40) 56.25 ft

42) process

44) 18285.7 miles/hr^2

or ≈ 7.45 ft/sec^2

5.2 Sigma Notation

The sum of n terms $a_1, a_2, a_3, \ldots, a_n$ is written as

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

where i is called the **index of summation,** a_i is called the ***i*th term** of the sum, and the **upper and lower bounds of summation** are n and 1, respectively.

Find: $\sum_{i=1}^{5} i^2 = 1 + 4 + 9 + 16 + 25$

$= 55$

Use sigma notation to write the given sum:

1 2 3 4 5 6

$$\frac{1}{2}+\frac{3}{2}+\frac{5}{2}+\frac{7}{2}+\frac{9}{2}+\frac{11}{2} = \frac{1}{2}(1+3+5+7\ldots)$$

$$\sum_{i=1}^{6} \frac{2i-1}{2} = \frac{1}{2}\sum_{i=1}^{6} 2i-1$$

Property:

$$\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i \text{ , where } k = \text{constant}$$

$$\sum_{i=1}^{5} 4 = \underline{4} + \underline{4} + \underline{4} + \underline{4} + \underline{4}$$

$$= 4(5)$$

Summation formulas:

1. $\sum_{i=1}^{n} c = cn$, where c = a constant

From the warm up:

$$1 + 2 + 3 + \ldots + 98 + 99 + 100 =$$

$$\begin{array}{cccccc} +100 & +99 & +98 & +3 & +2 & +1 \\ \hline 101 & 101 & 101 & 101 & 101 & 101 \end{array}$$

$$S_{100} = \frac{100(101)}{2} \rightarrow S_n = \frac{n(n+1)}{2}$$

Summation Formula

2. $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Summation Formulas: p. 240 Know these!!!

1. $\sum_{i=1}^{n} c = cn$
2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
4. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Evaluate the sum

$$\sum_{i=1}^{n} \frac{i+1}{n^2} = \sum_{i=1}^{10} \frac{i+1}{100}$$

for $n = 10$,

$$= \frac{1}{100}\left[\sum_{i=1}^{10} i + \sum_{i=1}^{10} 1\right]$$

$$= \frac{1}{100}\left[\frac{10(11)}{2} + 10\right]$$

$$= \frac{1}{100}(55 + 10)$$

$$= 0.65$$

Evaluate the sum

$$\sum_{i=1}^{n} \frac{i+1}{n^2}$$

for $n = 10$, 100, 1000, and $10{,}000$.

0.65

Summation Formula

2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$$\frac{1}{n^2}\sum_{i=1}^{n}(i+1)$$

$$\frac{1}{n^2}\left[\frac{n(n+1)}{2} + \frac{2n}{2}\right]$$

$$\frac{n^2+n+2n}{2n^2}$$

$$\frac{\cancel{n}(n+3)}{\cancel{n}(2n)}$$

n	Σ
10	0.65
100	0.515
1000	0.5015
10,000	0.50015
∞	

Let $s(n)$ be defined by

$$s(n) = \sum_{i=1}^{n}\left(2+\frac{i}{n}\right)^2\left(\frac{1}{n}\right)$$

and find the limit of $s(n)$ as $n \to \infty$.

$$S_n = \frac{1}{n}\sum\left(4 + \frac{4i}{n} + \frac{i^2}{n^2}\right)$$

$$= \frac{1}{n}\left[4n + \frac{4}{n}\sum i + \frac{1}{n^2}\sum i^2\right]$$

$$= \frac{1}{n}\left[4n + \frac{\cancel{4}^{2}}{\cancel{n}}\cdot\frac{\cancel{n}(n+1)}{\cancel{2}} + \frac{1}{n^{\cancel{2}}}\cdot\frac{\cancel{n}(n+1)(2n+1)}{6}\right]$$

$$= \frac{1}{n}\left[4n + 2n + 2 + \frac{2n^2+3n+1}{6n}\right]$$

$$= \frac{1}{n}\left(6n + 2 + \frac{n}{3} + \frac{1}{2} + \frac{1}{6n}\right)$$

$$S_n = 6 + \cancel{\frac{2}{n}}^{0} + \frac{1}{3} + \cancel{\frac{1}{2n}} + \cancel{\frac{1}{6n^2}}$$

$$\lim_{n\to\infty} S_n = 6 + \frac{1}{3}$$

$$= \frac{19}{3}$$

HW:

p. 248, # 1 - 25 odd

(skip 23)